Session 2: Stochastic Activity Network Concepts

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Session Goals

- Understand the workings and limitations of stochastic Petri nets.
- Understand the basics of stochastic activity networks, and how they work.
- Become familiar with looking at and understanding small SAN models.
- Understand reward structures and reward variables.
- Understand composed models and what they are appropriate for.
- Be prepared to build your first SAN model.
Session Outline

• Stochastic Petri nets
  – Places, tokens, input / output arcs, transitions
  – Readers / Writers example

• Stochastic activity networks
  – Input / output gates, cases, instantaneous and timed activities
  – Marking dependent behavior, well-specified, general distributions
  – Simple database server model

• Reward variables
  – Reward structures
  – Reward variable classification
  – Predicate / function implementation in Mobius

• Fault-tolerant computer example
• Composed models
  – Fault-tolerant computer revisited
Introduction

Stochastic activity networks, or SANs, are a convenient, graphical, high-level language for describing system behavior. SANs are useful in capturing the stochastic (or random) behavior of a system.

Examples:

- The amount of time a program takes to execute can be computed precisely if all factors are known, but this is nearly impossible and sometimes useless. At a more abstract level, we can approximate the running time by a random variable.

- Fault arrivals almost always must be modeled by a random process.

We begin by describing a subset of SANs: stochastic Petri nets.
Stochastic Petri Net Review

One of the simplest high-level modeling formalisms is called *stochastic Petri nets*. A stochastic Petri net is composed of the following components:

- **Places**: which contain tokens, and are like variables
- **Tokens**: which are the “value” or “state” of a place
- **Transitions**: which change the number of tokens in places
- **Input arcs**: which connect places to transitions
- **Output arcs**: which connect transitions to places
Firing Rules for SPNs

A stochastic Petri net (SPN) executes according to the following rules:

- A transition is said to be *enabled* if for each place connected by input arcs, the number of tokens in the place is $\geq$ the number of input arcs connecting the place and the transition.

Example:

Transition $t_1$ is enabled.
Firing Rules, cont.

• A transition may fire if it is enabled. (More about this later.)
• If a transition fires, for each input arc, a token is removed from the corresponding place, and for each output arc, a token is added to the corresponding place.

Example:

Note: tokens are not necessarily conserved when a transition fires.
Specification of Stochastic Behavior of an SPN

- A stochastic Petri net is made from a Petri net by
  - Assigning an exponentially distributed time to all transitions.
  - Time represents the “delay” between enabling and firing of a transition.
  - Transitions “execute” in parallel with independent delay distributions.

- Since the minimum of multiple independent exponentials is itself exponential, time between transition firings is exponential.

- If a transition $t$ becomes enabled, and before $t$ fires, some other transition fires and changes the state of the SPN such that $t$ is no longer enabled, then $t$ aborts, that is, $t$ will not fire.

- Since the exponential distribution is memoryless, one can say that transitions that remain enabled continue or restart, as is convenient, without changing the behavior of the network.
**SPN Example: Readers/Writers Problem**

There are at most $N$ requests in the system at a time. Read requests arrive at rate $\lambda_{ra}$, and write requests at rate $\lambda_{wa}$. Any number of readers may read from a file at a time, but only one writer may write at a time. A reader and writer may not access the file at the same time.

Locks are obtained with rate $\lambda_L$ (for both read and write locks); reads and writes are performed at rates $\lambda_r$ and $\lambda_w$ respectively. Locks are released at rate $\lambda_{rel}$.

Note:

\[ N \]

\[ \equiv \]

\( (N \text{ arcs}) \)
SPN Representation of Reader/Writers Problem
Notes on SPNs

- SPNs are much easier to read, write, modify, and debug than Markov chains.

- SPN to Markov chain conversion can be automated to afford numerical solutions to Markov chains.

- Most SPN formalisms include a special type of arc called an \textit{inhibitor arc}, which enables the SPN if there are zero tokens in the associated place, and the identity (do nothing) function. Example: modify SPN to give writes priority.

- Limited in their expressive power: may only perform $+$, $-$, $>$, and test-for-zero operations.

- These very limited operations make it very difficult to model complex interactions.

- Simplicity allows for certain analysis, e.g., a network protocol modeled by an SPN may detect deadlock (if inhibitor arcs are not used).

- More general and flexible formalisms are needed to represent real systems.
Stochastic Activity Networks

The need for more expressive modeling languages has led to several extensions to stochastic Petri nets. One extension that we will examine is called stochastic activity networks. Because there are a number of subtle distinctions relative to SPNs, stochastic activity networks use different words to describe ideas similar to those of SPNs.

Stochastic activity networks have the following properties:

- A general way to specify that an activity (transition) is enabled
- A general way to specify a completion (firing) rule
- A way to represent zero-timed events
- A way to represent probabilistic choices upon activity completion
- State-dependent parameter values
- General delay distributions on activities
Stochastic activity networks (hereafter SANs) have four new symbols in addition to those of SPNs:

- **Input gate**: used to define complex enabling predicates and completion functions

- **Output gate**: used to define complex completion functions

- **Cases**: (small circles on activities) used to specify probabilistic choices

- **Instantaneous activities**: used to specify zero-timed events
SAN Enabling Rules

An input gate has two components:

- enabling_function (state) $\rightarrow$ boolean; also called the \textit{enabling predicate}
- input_function(state) $\rightarrow$ state; rule for changing the state of the model

An activity is \textit{enabled} if for every connected input gate, the enabling predicate is true, and for each input arc, the number of tokens in the connected place $\geq$ number of arcs.

We use the notation $MARK(P)$ to denote the number of tokens in place $P$. 
Example SAN Enabling Rule

Example:

\[
\text{IG1 Predicate:}
\]

\[
\begin{align*}
&\text{if}((\text{MARK}(P1)>0 \ \&\& \ \text{MARK}(P2)==0) \ \| \\
&\quad (\text{MARK}(P1)==0 \ \&\& \ \text{MARK}(P2)>0)) \\
&\quad \text{return } 1; \\
&\quad \text{else return } 0;
\end{align*}
\]

Activity \textit{a1} is enabled if \textit{IG1} predicate is true (1) and \textit{MARK}(P3) > 0.
(Note that in \textit{Mobius}, “1” is used to denote true.)
Cases

Cases represent a probabilistic choice of an action to take when an activity completes.

When activity $a$ completes, a token is removed from place $P1$, and with probability $\alpha$, a token is put into place $P2$, and with probability $1 - \alpha$, a token is put into place $P3$.

Note: cases are numbered, starting with 1, from top to bottom.
Output Gates

When an activity completes, an output gate allows for a more general change in the state of the system. This output gate function is usually expressed using pseudo-C code.

Example

\[
\text{OG Function} \\
\text{MARK(P) = 0;}
\]
Another important feature of SANs is the instantaneous activity. An *instantaneous activity* is like a normal activity except that it completes in zero time after it becomes enabled. Instantaneous activities can be used with input gates, output gates, and cases.

Instantaneous activities are useful when modeling events that have an effect on the state of the system, but happen in negligible time, with respect to other activities in the system, and the performance/dependability measures of interest.
**SAN Terms**

1. *activation* - time at which an activity *begins*

2. *completion* - time at which activity *completes*

3. *abort* - time, after activation but before completion, when activity is no longer enabled

4. *active* - the time after an activity has been activated but before it completes or aborts.
Completion Rules

When an activity *completes*, the following events take place (in the order listed), possibly changing the marking of the network:

1. If the activity has cases, a case is (probabilistically) chosen.

2. The functions of all the connected input gates are executed (in an unspecified order).

3. Tokens are removed from places connected by input arcs.

4. The functions of all the output gates connected to the chosen case are executed (in an unspecified order).

5. Tokens are added to places connected by output arcs connected to the chosen case.

Ordering is important, since effect of actions can be marking-dependent.
Marking Dependent Behavior

Virtually every parameter may be any function of the state of the model. Examples of these are

- rates of exponential activities
- parameters of other activity distributions
- case probabilities

An example of this usefulness is a model of three redundant computers where the coverage (probability that a single computer crashing crashes the whole system) increases after a failure.

<table>
<thead>
<tr>
<th>Case</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 1</td>
<td>(0.1 + 0.02 \times \text{MARK}(P))</td>
</tr>
<tr>
<td>case 2</td>
<td>(0.9 - 0.02 \times \text{MARK}(P))</td>
</tr>
</tbody>
</table>
Example Problem

- A database server is composed of a compute server and three file servers, and can queue up to $N_c$ requests at a time (including the one in service).
- Requests arrive at rate $\lambda_a$ and spend on average $1/\lambda_{CPU}$ time at the compute server being processed.
- The request is then forwarded to the file server that has the fewest outstanding requests.
- Requests are processed at a rate of $\lambda_{D1}$, $\lambda_{D2}$, and $\lambda_{D3}$ for file servers D1, D2, and D3 respectively.
- File server buffers may hold at most $N_f$ requests (including requests in service); if all buffers are full, the request is discarded.
SAN Representation of Example Database Problem

Guard

\( \lambda_a \)

Arrival

Queue

CPU

Route

D1

D2

D3

\( \lambda_{D1} \)

\( \lambda_{D2} \)

\( \lambda_{D3} \)
### Gate Functions for SAN

<table>
<thead>
<tr>
<th>Gate</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guard</td>
<td><strong>Predicate</strong></td>
</tr>
<tr>
<td></td>
<td>( MARK(Queue) &lt; GLOBAL_S(Nc) )</td>
</tr>
<tr>
<td></td>
<td><strong>Function</strong></td>
</tr>
<tr>
<td>Route</td>
<td><strong>Function</strong></td>
</tr>
</tbody>
</table>
|       | \( \text{if } (MARK(D1) == GLOBAL_S(Nf) \text{ and } } \\  
|       | \( MARK(D2) == GLOBAL_S(Nf) \text{ and } } \\  
|       | \( MARK(D3) == GLOBAL_S(Nf) ) \text{ return; } \\  
|       | \( \text{if } (MARK(D1) < MARK(D2)) \{ \\  
|       | \( \text{if } (MARK(D1) < MARK(D3)) \{ \\  
|       | \( MARK(D1)+++; \) \\  
|       | \} \text{ else } \{ \\  
|       | \( MARK(D3)+++; \) \\  
|       | \} \\  
|       | \} \text{ else } \{ \\  
|       | \( \text{if } (MARK(D2) < MARK(D3)) \{ \\  
|       | \( MARK(D2)+++; \) \\  
|       | \} \text{ else } \{ \\  
|       | \( MARK(D3)+++; \) \\  
|       | \} \\  
|       | \} \

Stochastic Activity Network Concepts, in Mobius: Modeling, Analyzing, and Simulating Large-Scale Systems, Instructor William H. Sanders. ©2006 William H. Sanders. All rights reserved.
General Delay Distributions

Frequently, an exponential delay distribution is an acceptable approximation, but when more accuracy is required (and available), you may want to model using different delay distributions.

Example: The life of a rechargeable battery is often modeled using a Weibull distribution.

\[ F_X(t) = 1 - e^{-\left(\frac{t}{\beta}\right)^\alpha} \]

where parameters \( \alpha \) and \( \beta \) are found through measurement.

Recall: If an activity becomes disabled, when it later becomes enabled, it will start over.
General Delay Distributions, cont.

- SANs (and their implementation in *Mobius*) support many activity time distributions, including:
  - Exponential
  - Hyperexponential
  - Deterministic
  - Weibull
  - Conditional Weibull
  - Normal
  - Erlang
  - Gamma
  - Beta
  - Uniform
  - Binomial
  - Negative Binomial

- All distribution parameters can be marking-dependent
- The obvious implication of general delay distributions is that there is no conversion to a CTMC. Hence, no solutions to CTMCs are applicable. However, simulation is still possible.
- Analytical/numerical solution is possible for certain mixes of exponential and deterministic activities. See the *Mobius* manual for details.
- See [Kececioglu 91], for example, for appropriate use of some of these distributions.
Reward Variables

Reward variables are a way of measuring performance- or dependability-related characteristics about a model.

Examples:

- Expected time until service
- System availability
- Number of misrouted packets in an interval of time
- Processor utilization
- Length of downtime
- Operational cost
- Module or system reliability
Reward Structures

Reward may be “accumulated” two different ways:

- A model may be in a certain state or states for some period of time, for example, “CPU idle” states. This is called a rate reward.
- An activity may complete. This is called an impulse reward.

The reward variable is the sum of the rate reward and the impulse reward structures.
Reward Structure Example

A web server failure model is used to predict profits. When the web server is fully operational, profits accumulate at $N$/hour. In a degraded mode, profits accumulate at $\frac{1}{6}N$/hour. Repairs cost $K$.

$$R(m) = \begin{cases} 
N & \text{if } m \text{ is a fully functioning marking} \\
\frac{1}{6}N & \text{if } m \text{ is a degraded-mode marking} \\
0 & \text{otherwise}
\end{cases}$$

$$C(a) = \begin{cases} 
-K & \text{if } a \text{ is an activity representing repair} \\
0 & \text{otherwise}
\end{cases}$$

By carefully integrating the reward structure from 0 to $t$, we get the profit at time $t$. This is an example of an “interval-of-time” variable.
Reward Variables

A *reward variable* is the sum of the impulse and rate reward structures over a certain time.

Let \([t, t + l]\) be the interval of time defined for a reward variable:

- If \(l = 0\), then the reward variable is called an *instant-of-time* reward variable.
- If \(l > 0\), then the reward variable is called an *interval-of-time* reward variable.
- If \(l > 0\), then dividing an interval-of-time reward variable by \(l\) gives a *time-averaged interval-of-time* reward variable.
**Reward Variable Specification**

- **Instant-of-Time**
  - \( t \) lim as \( t \) goes to infinity

- **Time-Average Interval-of-Time**
  - \([t, t + l]\) lim as \( t \) goes to infinity

- **Interval-of-Time**
  - \([t, t + l]\) lim as \( l \) goes to infinity
Reward Variables are Random Variables

Note that since the behavior of a SAN is a stochastic process, then a reward variable is a measure defined on the stochastic process, and therefore a reward variable is a random variable.

A tool can solve for the reward variables, but solving for the distribution in many cases can be difficult. It is often much simpler to solve for the mean or variance of the reward variable, especially when using numerical techniques.

Example reward variables:

\[ A(0,t) \] - Fraction of time the system delivers proper service during \([0,t]\). Hard to compute.

\[ E[A(0,t)] \] - Expected value of \(A(0,t)\). Easier to compute.
Specifying Reward Variables in *Mobius*

- When specifying a rate portion of a reward structure in *Mobius*, you must define a predicate and function.
  - **predicate**: while true (i.e., integer greater than 0 in C), accumulate the reward
  - **function**: the value (i.e., double in C) to accumulate
- Note that both the predicate and function may be any C statement or expression.

### Queue Length

<table>
<thead>
<tr>
<th>Rate rewards</th>
<th>subnet = database</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicate</td>
<td>1</td>
</tr>
<tr>
<td>Function</td>
<td>MARK(Queue)</td>
</tr>
</tbody>
</table>

| Impulse reward | none |

Fault-Tolerant Computer Failure Model Example

A fault-tolerant computer system is made up of two redundant computers. Each computer is composed of three redundant CPU boards. A computer is operational if at least 1 CPU board is operational, and the system is operational if at least 1 computer is operational.

CPU boards fail at a rate of $1/10^6$ hours, and there is a 0.5% chance that a board failure will cause a computer failure, and a 0.8% chance that a board will fail in a way that causes a catastrophic system failure.
SAN computer for Computer Failure Model
## Activity Case Probabilities and Input Gate Definition

<table>
<thead>
<tr>
<th>Activity</th>
<th>Case</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPUfail1</td>
<td>1</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.008</td>
</tr>
</tbody>
</table>

### Gate Definition

<table>
<thead>
<tr>
<th>Gate</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enabled1</td>
<td><strong>Predicate</strong></td>
</tr>
<tr>
<td></td>
<td>( \text{MARK}(\text{CPUboards1} &gt; 0) &amp;&amp; \text{MARK}(\text{NumComp}) &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td><strong>Function</strong></td>
</tr>
<tr>
<td></td>
<td>( \text{MARK}(\text{CPUboards1}) - - ; )</td>
</tr>
</tbody>
</table>
### Output Gate Definitions

<table>
<thead>
<tr>
<th>Gate</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Covered1</strong></td>
<td>Function</td>
</tr>
<tr>
<td></td>
<td>( \text{if } (\text{MARK}(\text{CPUboards1}) == 0) )</td>
</tr>
<tr>
<td></td>
<td>( \text{MARK}(\text{NumComp})--; )</td>
</tr>
<tr>
<td><strong>Uncovered1</strong></td>
<td>Function</td>
</tr>
<tr>
<td></td>
<td>( \text{MARK}(\text{CPUboards1}) = 0; )</td>
</tr>
<tr>
<td></td>
<td>( \text{MARK}(\text{NumComp})--; )</td>
</tr>
<tr>
<td><strong>Catastrophic1</strong></td>
<td>Function</td>
</tr>
<tr>
<td></td>
<td>( \text{MARK}(\text{CPUboards1}) = 0; )</td>
</tr>
<tr>
<td></td>
<td>( \text{MARK}(\text{NumComp}) = 0; )</td>
</tr>
</tbody>
</table>
## Reward Variables for Computer Failure Model

<table>
<thead>
<tr>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rate rewards</strong></td>
</tr>
<tr>
<td><em>Subnet = computer</em></td>
</tr>
<tr>
<td><strong>Predicate:</strong></td>
</tr>
<tr>
<td>( \text{MARK}(\text{NumComp}) &gt; 0 )</td>
</tr>
<tr>
<td><strong>Function:</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td><strong>Impulse reward</strong></td>
</tr>
<tr>
<td>none</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NumBoardFailures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rate reward</strong></td>
</tr>
<tr>
<td>none</td>
</tr>
<tr>
<td><strong>Impulse reward</strong></td>
</tr>
<tr>
<td><em>Subnet = computer</em></td>
</tr>
<tr>
<td>activity = CPUfail1, value = 1</td>
</tr>
<tr>
<td>activity = CPUfail2, value = 1</td>
</tr>
</tbody>
</table>
## Reward Variables for Computer Failure Model

<table>
<thead>
<tr>
<th>Performability</th>
<th>Rate rewards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subnet = computer</td>
<td></td>
</tr>
<tr>
<td>Predicate:</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Function:</td>
<td></td>
</tr>
<tr>
<td>$MARK(\text{NumComp})$</td>
<td></td>
</tr>
<tr>
<td>Impulse reward</td>
<td></td>
</tr>
<tr>
<td>none</td>
<td></td>
</tr>
</tbody>
</table>

| NumBoards |
| Rate reward |
| Subnet = computer |
| Predicate: |
| 1 |
| Function: |
| $MARK(\text{CPUboards1}) + MARK(\text{CPUboards2})$ |
| Impulse reward |
| none |
A composed model is a way of connecting different SANs together to form a larger model.

Model composition has two operations:

– Replicate: Combine 2 or more identical SANs and reward structures together, holding certain places common among the replicas.

– Join: Combine 2 or more different SANs and reward structures together, combining certain places to permit communication.
Composed Model Specification

- Replicate submodel a certain number of times
- Hold certain places common to all replicas
- Join two or more submodels together
- Certain places in different submodels can be made common
Rationale

There are many good reasons for using composed models.

- Building highly reliable systems usually involves redundancy. The replicate operation models redundancy in a natural way.

- Systems are usually built in a modular way. Replicates and Joins are usually good for connecting together similar and different modules.

- Tools can take advantage of something called the Strong Lumping Theorem that allows a tool to generate a Markov process with a smaller state space (to be described in Session 7).
Rules for Building Composed Models

(as Implemented in *Mobius*)

- Places that are joined together must have the same name and initial marking.
- Places that are common at a certain level of the tree must be common at all lower levels.
- Places that are common cannot be connected to the input side of an instantaneous activity.
(Note initial marking of NumComp is two since there will be two computers in the composed model.)
Composed Model for Computer Failure Model

<table>
<thead>
<tr>
<th>Node</th>
<th>Reps</th>
<th>Common Places</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rep1</td>
<td>2</td>
<td>NumComp</td>
</tr>
</tbody>
</table>

Rep1

Computer
## Reward Variables for Composed Model

<table>
<thead>
<tr>
<th>Reliability</th>
<th>Rate rewards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subnet = computer</td>
<td></td>
</tr>
<tr>
<td>Predicate: MARK(NumComp) &gt; 0</td>
<td></td>
</tr>
<tr>
<td>Function: 0.5</td>
<td></td>
</tr>
<tr>
<td>Impulse reward</td>
<td></td>
</tr>
<tr>
<td>none</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NumBoardFailures</th>
<th>Rate reward</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td></td>
</tr>
<tr>
<td>Impulse reward</td>
<td></td>
</tr>
<tr>
<td>Subnet = computer</td>
<td></td>
</tr>
<tr>
<td>activity = CPUfail, value = 1</td>
<td></td>
</tr>
</tbody>
</table>
### Reward Variables for Composed Model

<table>
<thead>
<tr>
<th>Performability</th>
</tr>
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<tbody>
<tr>
<td><strong>Rate rewards</strong></td>
</tr>
<tr>
<td><em>Subnet = computer</em></td>
</tr>
<tr>
<td><strong>Predicate:</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td><strong>Function:</strong></td>
</tr>
<tr>
<td>( \frac{MARK(NumComp)}{2.0} )</td>
</tr>
<tr>
<td><strong>Impulse reward</strong></td>
</tr>
<tr>
<td>none</td>
</tr>
<tr>
<td><strong>NumBoards</strong></td>
</tr>
<tr>
<td><strong>Rate reward</strong></td>
</tr>
<tr>
<td><em>Subnet = computer</em></td>
</tr>
<tr>
<td><strong>Predicate:</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td><strong>Function:</strong></td>
</tr>
<tr>
<td>( MARK(CPUboards) )</td>
</tr>
<tr>
<td><strong>Impulse reward</strong></td>
</tr>
<tr>
<td>none</td>
</tr>
</tbody>
</table>
Composed Model

How does adding an additional computer affect reliability?

- In the composed model, change number of replications to 3 and change various reward variables - easy (Use a global variable if you think suspect you may want to do this.)
- In “flat” model, add another computer - hard

In composed model, the number of states in the underlying Markov chain is much smaller, especially for large numbers of replications. (Details will be given in Session 7.)
Global Variables

• There may be some parameter to a model that may want to change (in our example, the level of redundancy). This can be implemented easily as global variables.

• Global variables come as two types: short (short integer) and double (double precision floating point). Use `name` and `name` to indicate global variable “name.” (You do NOT need to declare these anywhere; UltraSAN does this automatically.)

• In Mobius, you may use global variables virtually anywhere you may type text.

Example: Use `num_comp` and `num_boards` to set the level of redundancy of computers and boards. Use this as the initial number of tokens, the number of replications, and when specifying reward variables. Later, you can change the levels of redundancy easily.
Summary

• Stochastic Petri nets
  – Places, tokens, input / output arcs
  – Readers / Writers example
• Stochastic activity networks
  – Input / output gates, cases, instantaneous activities
  – Marking dependent behavior, well-specified, general distributions
• Reward variables
  – Reward structures
  – Reward variable classification
  – Predicate / function implementation in UltraSAN
• Fault tolerant computer example
• Composed model
  – Fault tolerant computer revisited